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LETTER TO THE EDITOR

The asymptotic nature of isolated systems

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Abstract. It is shown that the retarded Maxwell field produced by a flat space–time isolated system consisting of two mutually interacting charged particles does not peel in the asymptotic past.

Since Penrose (1965) first introduced the concept of null asymptotic infinity it has often been tacitly assumed that an isolated solution of the Einstein–Maxwell equations M (i.e. a solution which is meant to describe a physically isolated system) should be characterised by the following asymptotic conditions.

(1) Future and past null infinities $\mathscr{I}^+$ and $\mathscr{I}^-$ exist and are smoothly attached to the conformally rescaled manifold $\hat{M}$.
(2) There is no incoming radiation in the sense that the radiation components of the conformally rescaled Weyl and Maxwell fields vanish on $\mathscr{I}^-$.

In terms of the standard spin-coefficient notation of Newman and Penrose (1962) condition (1) implies, in particular, that

$$\begin{aligned} \psi_0(v, r, \zeta, \bar{\zeta}) &= \psi_0^0(v, \zeta, \bar{\zeta})/r^5 + \mathrm{O}(r^{-6}) \\ \phi_0(v, r, \zeta, \bar{\zeta}) &= \phi_0^0(v, \zeta, \bar{\zeta})/r^3 + \mathrm{O}(r^{-4}) \end{aligned} \tag{1}$$

for any advanced Bondi-type coordinate system $(v, r, \zeta, \bar{\zeta})$.

Recent work by Walker and Will (1979) and Stewart (1979), however, suggests that for a large class of physically interesting systems (e.g. a two-body system) equations (1) may well be violated, in other words the fields may not peel in the asymptotic past.

The purpose of this Letter is to cast further doubt on the validity of equations (1) for isolated systems by showing that the retarded field produced by a flat space–time isolated system consisting of two mutually interacting charged particles does not peel in the asymptotic past. Since such a system is presumably the flat space correspondence limit of an associated general relativistic system, this result makes it highly implausible that equations (1) should be taken as a characteristic property of a GR isolated system.

Consider an isolated flat space–time system consisting of two particles of mass m_i and charge e_i $(i = 1, 2)$ whose world lines are given by $x^a = y_i^a(\tau_i)$, where τ_i is proper time. According to the Lorentz–Dirac force law (Rohrlich 1965) the equations of motion for the two particles are given by

$$\begin{aligned} m_1\ddot{y}_1^a &= e_1 F_{(2)}^{ab}\dot{y}_{1b} + R_1^a \\ m_2\ddot{y}_2^a &= e_2 F_{(1)}^{ab}\dot{y}_{2b} + R_2^a \end{aligned} \tag{2}$$

where $F_{(i)}^{ab}$ is the retarded field produced by particle (i), and R_i^a is the usual radiation reaction term. In order to avoid non-physical run-away solutions, these equations must be supplemented by the asymptotic condition

$$\ddot{y}_i^a \to 0 \qquad \text{as} \qquad \tau_i \to -\infty. \tag{3}$$

It would of course be an extremely difficult task to obtain an exact solution of these equations; nevertheless, it is relatively easy to obtain the first three terms in its asymptotic expansion for large negative τ_i. The result is

$$y_i^a \sim v_i^a \tau + A_i^a \ln(-\tau_i) + B_i^a \tag{4}$$

where

$$v_i^a v_{ia} = 1$$

$$A_1^a = 2e_1e_2 v_1^{[a} v_2^{b]} v_{1b}/m_1\alpha \tag{5}$$

$$A_2^a = 2e_1e_2 v_2^{[a} v_1^{b]} v_{2b}/m_2\alpha \tag{6}$$

and

$$\alpha = \{(v_1^a v_{2b})^2 - 1\}^{3/2}.$$

As we shall see, the presence of the log term in equation (4) prevents the associated Maxwell field $F_{(i)}^{ab}$ from peeling in the asymptotic past.

An advanced Bondi-type coordinate system $(v, r, \zeta, \bar{\zeta})$ can be defined as follows:

$$x^a = -r\mathscr{L}^a(\zeta, \bar{\zeta}) + V^a v$$

where

$$\mathscr{L}^a = [\sqrt{2}(1+\zeta\bar{\zeta})]^{-1}[1+\zeta\bar{\zeta}, \zeta+\bar{\zeta}, \mathrm{i}(\zeta-\bar{\zeta}), -1+\zeta\bar{\zeta}]$$

and

$$V^a = \sqrt{2}\delta_0^a.$$

The surfaces v = constant are past light cones whose null generators are labelled by the stereographic coordinate ζ; r, which is presumed to be positive, is an affine parameter along these generators ($r = \infty$ at $\mathscr{I}^-$). At any point whose coordinates are $(v, r, \zeta, \bar{\zeta})$, an associated null tetrad frame is given by

$$l^a = -\mathscr{L}^a, \qquad n^a = -(\mathscr{L}^a + \eth\bar{\eth}\mathscr{L}^a)$$

$$m^a = \eth l^a, \qquad \bar{m}^a = \bar{\eth} l^a$$

where $\eth$ and $\bar{\eth}$ are the standard spin-weighted differential operators of Newman and Penrose (1966).

It is easily seen that for a point with large r, the associated retarded times τ_i are large and negative. We may therefore use equation (4) to obtain the first two terms in the asymptotic expansion of $F_{(i)}^{ab}$ for large r. After a relatively simple calculation this yields

$$-F_{(i)}^{ab} \sim \frac{2e_i v_i^{[a} l^{b]}}{r^2 V_i^2} + \frac{e_i \ln r}{r^2 V_i^2}\left(\frac{6A_i v_i^{[a} l^{b]}}{V_i^2} + 2v_i^{[a} A_i^{b]}\right)$$

where $V_i = v_i^a l_a$ and $A_i = A_i^a l_a$.

This equation, together with equations (5) and (6), now yields

$$\phi_0 \overset{\text{def}}{=} F^{ab} l_a m_b = (F^{ab}_{(1)} + F^{ab}_{(2)}) l_a m_b \sim -\frac{2 \ln r}{r^3} \frac{e_1 e_2}{\alpha} \left(\frac{e_1}{m_1 V_1^3} - \frac{e_2}{m_2 V_2^3} \right) v_1^{[a} v_2^{b]} l_a m_b.$$

The total retarded field F^{ab} produced by the system therefore does not peel, in other words ϕ_0 does not satisfy

$$\phi_0 = \phi_0^0 / r^3 + \mathrm{O}(r^{-4}).$$

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